

Real-time correlators in Kerr/CFT correspondence

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ABSTRACT: We study real-time correlators in the Kerr/CFT correspondence. The near-horizon geometry of extreme and near-extremal Kerr black holes could be taken as the warped AdS spacetimes with the warping factor being a function of angular variable θ . We show that for the perturbations whose equations of motions could be decomposed into the angular part and radial part, their real-time correlation functions could be computed from warped AdS/CFT correspondence. We find that the retarded Green's functions, the cross sections and the quasi-normal modes are all in perfect match with the dual CFT predictions. The same analysis is also generalized to the charged Newman-Kerr black holes.

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1. Introduction

Much of the physics on black hole is encoded in the near-horizon geometry of the black hole. For example, the entropy of the black hole is just proportional to the area of the black hole horizon. And the Hawking radiation of the black hole could be effectively understood from the anomaly cancellation of the field theory in the near horizon geometry [1, 2]. A recent evidence to this universal property is the Kerr/CFT correspondence [3].

The Kerr/CFT correspondence conjectures that the quantum gravity in the near-horizon extreme Kerr (NHEK) geometry with certain boundary conditions is dual to a (1+1) dimensional chiral conformal field theory (CFT) ¹. The correspondence was inspired by the properties of the asymptotic symmetry group of the near horizon geometry [4] of the extreme Kerr black hole where it was found by Guica, Hartman, Song and Strominger (GHSS) [3] that under a certain set of boundary condition on the asymptotic behaviour of the metric, the $U(1)_L$ symmetry of the $SL(2, R)_R \times U(1)_L$ isometry group [5] of the near-horizon geometry get enhanced into a Virasoro algebra. Support of this conjecture has been found in the perfect match of the macroscopic Berenstein-Hawking entropy of the black hole with the conformal field theory entropy computed by the Cardy formula. See [6] for some further studies of the Kerr/CFT correspondence as well as generalizations to other spacetime which contain a warped AdS structure.

¹More precisely, the Kerr/CFT correspondence can be referred to as the NHEK/CFT correspondence.

Further support of the correspondence are found in the studies of the superradiant scattering processes off extreme Kerr black holes [7]. In this case, the near horizon geometry of a near-extremal Kerr black hole (near-NHEK) is reminiscent of a non-extremal warped black hole. Correspondingly, the right-moving sector of dual CFT is excited [8]. An important ingredient in the studies is the Teukolsky master equations [9, 10, 11, 12, 13], which include an angular equation and a radial equation. In the near-horizon limit, the modes of interest are the ones near the super-radiant bound. This implies that the separation constant is well independent of the frequency of the mode and so the radial equation can be decoupled completely from the angular equation. This allows [7] to compute the quantum decay rate of the bulk fields and the greybody factor of the extreme Kerr black hole. On the CFT side, the decay rate and the absorption cross section can be extracted from the two-point correlation function [14]. It is remarkable that the bulk scattering results are in precise agreement with the CFT description whose form is completely fixed by conformal invariance. Similar discussion has been generalized to charged Kerr-Newman [15], multi-charged Kerr [16] and higher dimensional near-extremal Kerr black holes. In all these cases, perfect agreement with the dual CFT description has been found.

Note that in these tests of the Kerr/CFT correspondence, the fundamental CFT two-point correlator is compared with secondary quantities such as the decay rates derived from the superradiant scattering processes. We recall that in the standard AdS/CFT correspondence, it is possible to extract the CFT real-time correlator directly from the bulk asymptotic AdS spacetime [17]. This prescription has also been shown to work [18] for the warped AdS/CFT correspondence [19] as well ². It is natural to ask if one can also compute the real-time correlators directly from the bulk side for the Kerr/CFT correspondence. This will allow one to perform a test directly on the CFT correlators and the real-time correlators as obtained by holography. Now although the NHEK geometry is more complicated, it is in fact a warped AdS₃ spacetime with a warping factor being a function of the angular variable, therefore one can consider the Kerr/CFT correspondence as a generalization of the warped AdS/CFT correspondence. In this paper, we show that with a small modification, the Minkowskian prescription for computing the real-time correlators continues to work. The results are in perfect agreement with the CFT predictions.

In the next section, we give a brief review of Kerr/CFT correspondence for extreme and near-extremal Kerr black holes. In section 3, we outline the forms of the correlation functions as determined by the conformal invariance in 2D conformal field theory. In section 4 and 5, by considering the Kerr/CFT correspondence as a generalized warped AdS/CFT correspondence, we use the Minkowskian prescription to compute retarded correlators of various fields in near-NHEK and NHEK case respectively. We will show that the results agree precisely with the CFT correlators provided that one absorb away an angular-dependent multiplicative factor due to the angular dependent warping factor. In section 6, we discuss the charged Kerr-Newman case. We end with some discussions in section 7.

²For a discussion of other related issues of the warped AdS/CFT correspondence, see for example [20, 21, 22, 23, 24, 25].

2. Brief Review of the Kerr/CFT Correspondence

A Kerr black hole is characterized by the mass M and angular momentum $J = aM$. It could be described by the metric of the following form

$$ds^2 = -\frac{\Delta}{\hat{\rho}^2}(d\hat{t} - a \sin^2 \theta d\hat{\phi})^2 + \frac{\sin^2 \theta}{\hat{\rho}^2} \left((\hat{r}^2 + a^2) d\hat{\phi} - a d\hat{t} \right)^2 + \frac{\hat{\rho}^2}{\Delta} d\hat{r}^2 + \hat{\rho}^2 d\theta^2, \quad (2.1)$$

with

$$\Delta = \hat{r}^2 - 2M\hat{r} + a^2, \quad \hat{\rho}^2 = \hat{r}^2 + a^2 \cos^2 \theta, \quad (2.2)$$

where we have used the unit $G = \hbar = c = 1$. In general, there are two horizons at $\Delta = 0$, which gives

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}. \quad (2.3)$$

The Hawking temperature, the angular velocity of the horizon and the entropy of the Kerr black hole are

$$T_H = \frac{r_+ - r_-}{8\pi M r_+}, \quad \Omega_H = \frac{a}{2Mr_+}, \quad S_{BH} = 2\pi M r_+. \quad (2.4)$$

For the extreme Kerr black hole, the Hawking temperature is zero, and its entropy is

$$S_{\text{ext}} = 2\pi J = 2\pi M^2. \quad (2.5)$$

We are interested in the extreme and near-extremal Kerr black holes. First we focus on the near-extremal Kerr case, whose near horizon geometry could be defined by taking the limit $T_H \rightarrow 0$ and $\hat{r} \rightarrow r_+$ and with the dimensionless near-horizon temperature

$$T_R \equiv \frac{2MT_H}{\lambda} \quad (2.6)$$

being fixed when $\lambda \rightarrow 0$. In other words, even though the Hawking temperature at asymptotic infinity is zero, the temperature measured near the horizon is still finite due to the infinite blueshift. For the extreme black hole, T_R is exactly zero even at the horizon. As in [7], for the near-extremal Kerr, we have

$$r_+ = M + \lambda M 2\pi T_R + O(\lambda^2), \quad (2.7)$$

$$a = M - 2M(\lambda\pi T_R)^2 + O(\lambda^3). \quad (2.8)$$

After redefining the coordinates

$$t = \lambda \frac{\hat{t}}{2M}, \quad (2.9)$$

$$r = \frac{\hat{r} - r_+}{\lambda r_+}, \quad (2.10)$$

$$\phi = \hat{\phi} - \frac{\hat{t}}{2M}, \quad (2.11)$$

and keeping T_R fixed, we obtain the near-extremal near-horizon metric

$$ds^2 = 2J\Gamma \left(-r(r + 2\alpha)dt^2 + \frac{dr^2}{r(r + 2\alpha)} + d\theta^2 + \Lambda^2(d\phi + (r + \alpha)dt)^2 \right), \quad (2.12)$$

where $\alpha = 2\pi T_R$,

$$\Gamma(\theta) = \frac{1 + \cos^2 \theta}{2}, \quad \Lambda(\theta) = \frac{2 \sin \theta}{1 + \cos^2 \theta} \quad (2.13)$$

and $\phi \sim \phi + 2\pi, 0 \leq \theta \leq \pi$. For the extreme Kerr black hole, $T_R = 0$, its near-horizon geometry in Poincare-type coordinates is [5]

$$ds^2 = 2J\Gamma \left(-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2 (d\phi + r dt)^2 \right). \quad (2.14)$$

In global coordinates, the metric is

$$ds^2 = 2J\Gamma \left(-(1 + \rho^2) d\tau^2 + \frac{d\rho^2}{1 + \rho^2} + d\theta^2 + \Lambda^2 (d\varphi + \rho d\tau)^2 \right). \quad (2.15)$$

The NHEK geometry has an isometry $SL(2, R)_R \times U(1)_L$. Moreover, for each three-dimensional slice of fixed polar angle θ , (2.15) is a global warped AdS_3 spacetime, while the NHEK geometry (2.14) is the quotient of warped AdS_3 . In other words, just as the BTZ black hole as the quotient of AdS_3 [26], the global warped AdS spacetime (2.15) can be taken as the vacuum with the geometry (2.14) (resp. (2.12)) being taken as an extreme warped AdS_3 black hole (resp. as a non-extremal warped AdS_3 black hole). Note that although the metrics (2.14) and (2.15) can be related by the coordinate transformation, but they describe different geometries since the coordinate transformation is singular.

It was shown in [3] that with consistent boundary condition the $SL(2, R)_R$ becomes trivial while the $U(1)_L$ is enhanced to a Virasoro algebra with central charge $c_L = 12J$. Moreover, the quantum theory in the Frolov-Thorne vacuum for extreme Kerr has the left-moving temperature $T_L = \frac{1}{2\pi}$. Therefore the Cardy formula gives the entropy for the dual CFT

$$S = \frac{\pi^2}{3} c_L T_L = 2\pi J. \quad (2.16)$$

This matches exactly with the Bekenstein-Hawking entropy of the extreme Kerr black hole. It is important to emphasize that the Kerr/CFT correspondence is a correspondence between the quantum gravity in the NHEK geometry and a 2D chiral CFT with c_L and T_L . In fact as shown in [7], the CFT cross section only counts for the scattering amplitude near the horizon region rather than the whole extreme Kerr black hole including the asymptotic region.

For the near-extremal Kerr black hole, its near horizon geometry is (2.12). The entropy of the near-extremal Kerr black hole is the same as for the extreme one. Now in dual 2D CFT, the right-moving sector is excited with a finite temperature. The Cardy formula gives the entropy

$$S = \frac{\pi^2}{3} (c_L T_L + c_R T_R). \quad (2.17)$$

In order to match with the black hole entropy, the central charge in the right-moving sector should be zero. It will be interesting to find the consistent boundary conditions that lead to an asymptotic symmetry group which extends the $SL(2, R)_R$ to Virasoro and compute the central charge to confirm this.

3. Two-point Correlators in 2D CFT

In a 2D conformal field theory(CFT), one can define the two-point function

$$G(t^+, t^-) = \langle \mathcal{O}_\phi^\dagger(t^+, t^-) \mathcal{O}_\phi(0) \rangle, \quad (3.1)$$

where t^+, t^- are the left and right moving coordinates of 2D worldsheet and \mathcal{O}_ϕ is the operator corresponding to the field perturbing the black hole. For an operator of dimensions (h_L, h_R) , charges (q_L, q_R) at temperature (T_L, T_R) and chemical potentials (Ω_L, Ω_R) , the two-point function is dictated by conformal invariance and takes the form [27]:

$$G(t^+, t^-) \sim (-1)^{h_L+h_R} \left(\frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left(\frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R} e^{iq_L \Omega_L t^+ + iq_R \Omega_R t^-}. \quad (3.2)$$

In frequency domain, the decay rate and the CFT absorption cross section are given by [14]

$$\begin{aligned} \Gamma &\sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+} G(t^+ - i\epsilon, t^- - i\epsilon), \\ \sigma_{\text{abs}} &\sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+} [G(t^+ - i\epsilon, t^- - i\epsilon) - G(t^+ + i\epsilon, t^- + i\epsilon)]. \end{aligned} \quad (3.3)$$

It follows that

$$\Gamma \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{-\bar{\omega}_L/2T_L} e^{-\bar{\omega}_R/2T_R} |\Gamma(h_L + i\frac{\bar{\omega}_L}{2\pi T_L})|^2 |\Gamma(h_R + i\frac{\bar{\omega}_R}{2\pi T_R})|^2, \quad (3.4)$$

and $\sigma_{\text{abs}} \sim \Gamma(\bar{\omega}_L, \bar{\omega}_R) \pm \Gamma(-\bar{\omega}_L, -\bar{\omega}_R)$ giving

$$\sigma_{\text{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} |\Gamma(h_L + \frac{i\bar{\omega}_L}{2\pi T_L})|^2 |\Gamma(h_R + \frac{i\bar{\omega}_R}{2\pi T_R})|^2 \cdot \begin{cases} \sinh(\frac{\bar{\omega}_L}{2T_L} + \frac{\bar{\omega}_R}{2T_R}), & (\text{boson}) \\ \cosh(\frac{\bar{\omega}_L}{2T_L} + \frac{\bar{\omega}_R}{2T_R}). & (\text{fermion}) \end{cases} \quad (3.5)$$

Here $\bar{\omega}_L, \bar{\omega}_R$ are defined by

$$\bar{\omega}_L = \omega_L - q_L \Omega_L, \quad \bar{\omega}_R = \omega_R - q_R \Omega_R. \quad (3.6)$$

The \sim sign here (and below) means the LHS and the RHS are equal up to a factor independent of the frequencies. A perfect match has been found for the expressions (3.4), (3.5) with the macroscopic decay rate and the greybody factor for a bulk field scattering off (near)-NHEK [7]. This gives strong support to the Kerr/CFT correspondence.

Apart from the relations (3.4), (3.5), another connection of the two-point function with the bulk can be developed. Let us introduce the Euclidean correlator G_E by a Wick rotation $t^+ \rightarrow i\tau_L$, $t^- \rightarrow i\tau_R$. At finite temperature the Euclidean time is taken to have period $2\pi/T_L, 2\pi/T_R$ and the momentum space Euclidean correlator is given by

$$G_E(\omega_{L,E}, \omega_{R,E}) = \int_0^{2\pi/T_L} d\tau_L \int_0^{2\pi/T_R} d\tau_R e^{-i\omega_{L,E}\tau_L - i\omega_{R,E}\tau_R} G_E(\tau_L, \tau_R), \quad (3.7)$$

where the Euclidean frequencies are related to the Minkowskian ones by

$$\omega_{L,E} = i\omega_L, \quad \omega_{R,E} = i\omega_R. \quad (3.8)$$

The integral is divergent but can be defined by analytic continuation, one obtains [14]

$$G_E(\omega_{L,E}, \omega_{R,E}) \sim T_L^{2h_L-1} T_R^{2h_R-1} e^{i\frac{\bar{\omega}_{L,E}}{2T_L}} e^{i\frac{\bar{\omega}_{R,E}}{2T_R}} \Gamma(h_L + \frac{\bar{\omega}_{L,E}}{2\pi T_L}) \Gamma(h_L - \frac{\bar{\omega}_{L,E}}{2\pi T_L}) \cdot \Gamma(h_R + \frac{\bar{\omega}_{R,E}}{2\pi T_R}) \Gamma(h_R - \frac{\bar{\omega}_{R,E}}{2\pi T_R}), \quad (3.9)$$

where

$$\bar{\omega}_{L,E} = \omega_{L,E} - iq_L \Omega_L, \quad \bar{\omega}_{R,E} = \omega_{R,E} - iq_R \Omega_R. \quad (3.10)$$

We note that $G_E(\omega_{L,E}, \omega_{R,E})$ is related to the value of the retarded correlator $G_R(\omega_L, \omega_R)$. More specifically, the retarded correlator $G_R(\omega_L, \omega_R)$ is analytic on the upper half complex $\omega_{L,R}$ -plane and its value along the positive imaginary $\omega_{L,R}$ -axis gives the Euclidean correlator:

$$G_E(\omega_{L,E}, \omega_{R,E}) = G_R(i\omega_{L,E}, i\omega_{R,E}), \quad \omega_{L,E}, \omega_{R,E} > 0. \quad (3.11)$$

This relation holds both for zero and finite temperature. However at finite temperature, $\omega_{L,E}$ and $\omega_{R,E}$ take discrete values of the Matsubara frequencies

$$\omega_{L,E} = 2\pi m_L T_L, \quad \omega_{R,E} = 2\pi m_R T_R, \quad (3.12)$$

where m_L, m_R are integers for bosonic modes and are half integers for fermionic modes. As we mentioned in the introduction, since the retarded correlator can be computed directly with a holographic prescription in terms of the bulk, therefore (3.11) provides a direct test of the Kerr/CFT correspondence.

4. Real-time Correlators in Near-NHEK of Kerr Black Hole

In the AdS/CFT correspondence, one subtle point in the computation of real-time correlators is the boundary conditions of the classical solution at the black hole horizon. Different Green's functions correspond to different boundary conditions at the horizon. It turns out that the retarded Green's function corresponds to the ingoing boundary condition, while the advanced Green's function corresponds to the outgoing one. However, even after fixing the boundary condition, one cannot obtain the correlators by naively using the prescription in the Euclidean version of the AdS/CFT correspondence.

In [17], a simple prescription was proposed to compute real-time correlators from gravity. This prescription has been instrumental to the study of strongly interacting system at finite temperature during the past few years. It has also been justified from different points of view in [28, 29, 30, 31, 32, 33]. It was later observed [30] that this prescription could be recast in terms of the boundary values of the canonical conjugate momentum of the bulk fields by treating the AdS radial direction as “time” direction. Furthermore, this reformulation was shown to be able to follow directly from the analytic continuation of Euclidean AdS/CFT correspondence [33]. More precisely, for a background metric

$$ds^2 = g_{rr} dr^2 + g_{\mu\nu} dx^\mu dx^\nu, \quad (4.1)$$

where μ, ν run over a d -dimensional spacetime, assume that the metric has an event horizon at $r = r_0$ and a boundary at $r = \infty$. Also assume that all metric components depends only on r , then the prescription for computing the retarded correlator is:

$$G_R(\omega, \vec{k}) \sim \left(\lim_{r \rightarrow \infty} r^N \frac{\Pi(r, \omega, \vec{k})|_{\phi_R}}{\phi_R(r, \omega, \vec{k})} \right) \Big|_{\phi_0=0}, \quad (4.2)$$

where

$$\Pi = -\sqrt{-g} g^{rr} \partial_r \phi \quad (4.3)$$

is the canonical momentum conjugate to ϕ , taking r as the “time” direction, ϕ_R is a classical solution which should be purely in-falling at the black hole horizon and approaches to $\phi_0(\omega, \vec{k})$ asymptotically. In order for (4.2) to give a well-defined result independent of r , a certain factor r^N is inserted whose power depends on the asymptotic behaviour of the metric as well as the solution ϕ_R . For the standard AdS case, it is $N = 2(\Delta - d)$ where Δ is the conformal dimension of the operator \mathcal{O}_ϕ . Finally the subscript $\phi_0 = 0$ means that one should take the part that is independent of ϕ_0 .

It is remarkable that the right hand side of (4.2) needs proper holographic renormalization. Such a renormalization may affect the overall normalization of the two-point functions, and is well-understood in the context of usual AdS/CFT correspondence[31]. However, for the warped spacetimes, which are not asymptotical to AdS, the holographic renormalization procedure has not been analyzed carefully. In this sense, all the results based on (4.2) should be taken with care. Nevertheless, from the study of real-time correlators in the warped AdS/CFT correspondence we have learned that the prescription (4.2) has worked quite well[18]. It will be interesting to study the holographic renormalization in the warped AdS/CFT correspondence in details and confirm the prescription (4.2).

The prescription works well not just for asymptotic AdS metric, but also for the warped AdS/CFT correspondence [18]. For the Kerr/CFT correspondence, the NHEK or the near-NHEK geometry is a warped AdS₃ spacetime with the warping factor being a function of the angular variable θ . Since this modification is quite simple, the prescription (4.2) actually factorizes into a part which depends on θ only and a part which is a function of the frequencies. Therefore one can have a well-defined prescription by taking only the angular independent part. In general, it is the asymptotic behaviour of the classical solution and its conjugate momentum that matter in (4.2). For example, for a scalar field with the asymptotic behaviour

$$\phi \sim A(\omega, \vec{k}) r^{-n_A} + B(\omega, \vec{k}) r^{-n_B}, \quad (4.4)$$

with $n_A > n_B$, the real-time correlator of the scalar field is given by $A(\omega, \vec{k})/B(\omega, \vec{k})$, up to a constant factor independent of ω and \vec{k} which depends on the normalization of the operator. This simple result generalizes to other kinds of fields as we will show below.

4.1 Scalar field

Let us consider the scalar field Φ of mass μ in the background (2.12). Since there are two translational Killing vector along t and ϕ , we may take the ansatz:

$$\Phi = e^{-i\omega t + im\phi} \mathcal{R}(r) \mathcal{S}(\theta), \quad (4.5)$$

where ω and m are the quantum numbers. The angular part $\mathcal{S}(\theta)$ satisfies the spheroidal harmonic equation:

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \mathcal{S} \right) + \left(\Lambda_{lm} - \left(\frac{m^2}{4} - J\mu^2 \right) \sin^2 \theta - \frac{m^2}{\sin^2 \theta} \right) \mathcal{S} = 0, \quad (4.6)$$

where Λ_{lm} is the eigenvalue, which can be computed numerically. The radial part \mathcal{R} satisfies the equation

$$\frac{d}{dr} \left(r(r+2\alpha) \frac{d}{dr} \right) \mathcal{R}(r) - \left(\Lambda_{lm} - m^2 + 2J\mu^2 - \frac{(\omega + m(r+\alpha))^2}{r(r+2\alpha)} \right) \mathcal{R}(r) = 0. \quad (4.7)$$

Taking into account of the ingoing boundary condition at the horizon, the radial wave function is

$$\mathcal{R}(r) = N r^{-\frac{i}{2}(m+\frac{\omega}{\alpha})} \left(\frac{r}{2\alpha} + 1 \right)^{-\frac{i}{2}(m-\frac{\omega}{\alpha})} F \left(\frac{1}{2} + \beta - im, \frac{1}{2} - \beta - im, 1 - i(m + \frac{\omega}{\alpha}); -\frac{r}{2\alpha} \right), \quad (4.8)$$

where

$$\beta^2 = \frac{1}{4} + \Lambda_{lm} - 2m^2 + 2J\mu^2. \quad (4.9)$$

At asymptotic infinity, the radial eigenfunction has the behaviour

$$\mathcal{R}(r) \sim A r^{-\frac{1}{2}-\beta} + B r^{-\frac{1}{2}+\beta} \quad (4.10)$$

where

$$A = N \frac{\Gamma(-2\beta) \Gamma(1 - i(m + \frac{\omega}{\alpha}))}{\Gamma(\frac{1}{2} - \beta - im) \Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})} (2\alpha)^{\frac{1}{2}+\beta-\frac{i}{2}(m+\frac{\omega}{\alpha})}, \quad (4.11)$$

$$B = A(\beta \rightarrow -\beta) \quad (4.12)$$

and N is an arbitrary constant. Without loss of generality, let us consider a real $\beta > 0$, the prescription (4.2) gives the retarded correlator

$$\begin{aligned} G_R &\sim \frac{A}{B} \\ &= (2\alpha)^{2\beta} \frac{\Gamma(-2\beta)}{\Gamma(2\beta)} \frac{\Gamma(\frac{1}{2} + \beta - im) \Gamma(\frac{1}{2} + \beta - i\frac{\omega}{\alpha})}{\Gamma(\frac{1}{2} - \beta - im) \Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})}. \end{aligned} \quad (4.13)$$

By comparing the arguments of the Gamma functions, one have the following identification

$$h_L = h_R = \frac{1}{2} + \beta, \quad \omega_L = m, \quad \bar{\omega}_R = \omega, \quad T_L = \frac{1}{2\pi}, \quad T_R = T_R, \quad (4.14)$$

in order for (3.11) to be satisfied. In fact at the Matsubara frequencies, the expression (4.13) agrees precisely with (3.9) up to an irrelevant normalization factor which depends only on β , q_L and q_R and can be absorbed into the normalization of the operator. The identification (4.14) is the same as the original ones suggested in [7] where our ω is n_R in their notation. For imaginary β , the above formula stays the same. The complex conformal weight indicates an instability of the AdS spacetime due to pair production [7].

The cross section can also be read out from the retarded correlator directly. It is

$$\sigma = \text{Im}(G_R) = \frac{(2\alpha)^{2\beta}}{2\beta\pi(\Gamma(2\beta))^2} \sinh(\pi(m + \frac{\omega}{\alpha})) |\Gamma(\frac{1}{2} + \beta - im)\Gamma(\frac{1}{2} + \beta - i\frac{\omega}{\alpha})|^2. \quad (4.15)$$

This agree, up to an irrelevant normalization factor, with (3.5) as it should be.

Finally, one can obtain the quasi-normal modes frequencies from the poles of the retarded Green's function. In this case, they are

$$\begin{aligned} \bar{\omega}_L &= -i2\pi T_L(n_L + h_L) \\ \bar{\omega}_R &= -i2\pi T_R(n_R + h_R) \end{aligned} \quad (4.16)$$

with n_L, n_R being non-negative integers. The left part is not actually the quasi-normal modes since it is related to the quantum number of rotation. The right part gives the contribution. As a result, we obtain the complete frequencies of the quasi-normal modes:

$$\omega_R = m\Omega_H - i2\pi T_R(n_R + h_R). \quad (4.17)$$

This has also been obtained in [34] for the near-extreme Kerr black holes.

4.2 Other perturbations

To study various kinds perturbations about near-NHEK, including vector, spinor and gravitational ones, we will use Newman-Penrose formalism [35]. For simplicity, we focus on the massless perturbations. The NP null tetrad of near-NHEK is $e_a^\mu = (l^\mu, n^\mu, m^\mu, m^{*\mu})$, where in coordinate basis

$$\begin{aligned} l^\mu &= \frac{1}{r(r+2\alpha)}(1, r(r+2\alpha), 0, -(r+\alpha)), \\ n^\mu &= \frac{1}{4J\Gamma(\theta)}(1, -r(r+2\alpha), 0, -(r+\alpha)), \\ m^\mu &= \frac{1}{2\sqrt{J\Gamma(\theta)}}(0, 0, 1, i\Lambda^{-1}(\theta)), \end{aligned} \quad (4.18)$$

satisfy the normalization and orthogonal condition with nonvanishing inner products

$$l \cdot n = -m \cdot m^* = -1. \quad (4.19)$$

It turns out that the equations of motions of the perturbations can be decomposed into two separated equations of motions. The wave function could be decomposed into the form

$$\Psi^s = e^{-i\omega t + im\phi} (r(r+2\alpha))^{-s} \mathcal{R}^s(r) \mathcal{S}^s(\theta). \quad (4.20)$$

Here the angular function $\mathcal{S}^s(\theta)$ obeys the equation

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \mathcal{S}^s(\theta) \right) + \left(\Lambda_{lm}^s - \frac{m^2}{4} \sin^2\theta - ms \cos\theta - \frac{(m+s\cos\theta)^2}{\sin^2\theta} \right) \mathcal{S}^s(\theta) = 0, \quad (4.21)$$

where Λ_{lm}^s is the separation parameter, depending on l, m, s and satisfying

$$\Lambda_{lm}^s(s) = \Lambda_{lm}^s(-s). \quad (4.22)$$

The radial function obeys

$$\frac{d}{dr} \left(r(r+2\alpha) \frac{d}{dr} \right) \mathcal{R}^s(r) - \left(\Lambda_{lm}^s + q^2 - 2m^2 - \frac{(\omega + q(r+\alpha))^2}{r(r+2\alpha)} \right) \mathcal{R}^s(r) = 0. \quad (4.23)$$

where

$$q = m - is. \quad (4.24)$$

The ingoing solution is

$$\mathcal{R}^s(r) = N r^{-\frac{i}{2}(q+\frac{\omega}{\alpha})} \left(\frac{r}{2\alpha} + 1 \right)^{-\frac{i}{2}(q-\frac{\omega}{\alpha})} F \left(\frac{1}{2} + \beta - iq, \frac{1}{2} - \beta - iq, 1 - i(q + \frac{\omega}{\alpha}); -\frac{r}{2\alpha} \right), \quad (4.25)$$

where

$$\beta^2 = \frac{1}{4} + \Lambda_{lm}^s - 2m^2. \quad (4.26)$$

The asymptotic behaviour of the solution is

$$\mathcal{R}^s(r) \sim A^s r^{-\frac{1}{2}-\beta} + B^s r^{-\frac{1}{2}+\beta}, \quad (4.27)$$

where

$$A^s = N \frac{\Gamma(-2\beta)\Gamma(1-i(q+\frac{\omega}{\alpha}))}{\Gamma(\frac{1}{2}-\beta-iq)\Gamma(\frac{1}{2}-\beta-i\frac{\omega}{\alpha})} (2\alpha)^{\frac{1}{2}+\beta-\frac{i}{2}(q+\frac{\omega}{\alpha})} \quad (4.28)$$

$$B^s = A^s(\beta \rightarrow -\beta) \quad (4.29)$$

Naively one may be tempted to take the retarded Green's function to be proportional to $\frac{A^s}{B^s}$, as in scalar case. This is not true. In the usual AdS/CFT correspondence, the prescription to get the retarded Green's function is (4.2). In our case, things are more subtle. In fact, for $|s| = 1, 2$, Ψ^s are related to the gauge field strength and the Weyl tensor of the tensor field:

$$\begin{aligned} \Psi^1 &= F_{\mu\nu} l^\mu m^\nu \\ \Psi^{-1} &= (1 - i \cos \theta)^2 F_{\mu\nu} m^{*\mu} n^\nu \\ \Psi^2 &= C_{\mu\nu\rho\sigma} l^\mu m^\nu l^\rho m^\sigma \\ \Psi^{-2} &= (1 - i \cos \theta)^4 C_{\mu\nu\rho\sigma} n^\mu m^{*\nu} n^\rho m^{*\sigma} \end{aligned} \quad (4.30)$$

Therefore it is not appropriate to identify Ψ^s as the perturbations themselves. Nevertheless, we can inversely obtain the vector and gravitational perturbations from the wave functions [36] in terms of the Newman-Penrose complex spin coefficients

$$A_\mu = -(-l_\mu(\delta^* + 2\beta^* + \tau^*) + m_\mu^*(D + \rho^*)) \frac{2}{B_1} (r(r+2\alpha)) R^{-1}(r) \mathcal{S}^1(\theta) e^{-i\omega t + im\phi}, \quad (4.31)$$

$$\begin{aligned} h_{\mu\nu} &= \left\{ -l_\mu l_\nu (\delta^* + \lambda + 3\beta^* - \tau^*) (\delta^* + 4\beta^* + 3\tau^*) - m_\mu^* m_\nu^* (D - \rho^*) (D + 3\rho^*) \right. \\ &\quad \left. + l_{(\mu} m_{\nu)}^* ((D + \rho - \rho^*) (\delta^* + 4\beta^* + 3\tau^*) + (\delta^* + 3\beta^* - \lambda - \pi - \tau^*) (D + 3\rho^*)) \right\} \\ &\quad \times \frac{4}{B_2} (r(r+2\alpha))^2 R^{-2}(r) \mathcal{S}^2(\theta) e^{-i\omega t + im\phi}, \end{aligned} \quad (4.32)$$

where B_1 and B_2 are two normalization factors depending on Λ_{lm} and m . In the above relations, the differential operators D and δ^* are defined as

$$D = l^\mu \partial_\mu, \quad \delta^* = m^{*\mu} \partial_\mu, \quad (4.33)$$

and the spin coefficients are

$$\begin{aligned} \rho &= -\frac{1}{1 - i \cos \theta}, & \beta &= \frac{1}{2\sqrt{2J}} \frac{\cos \theta}{(1 + i \cos \theta) \sin \theta}, \\ \tau &= -\frac{1}{\sqrt{2J}} \frac{i \sin \theta}{1 + \cos^2 \theta}, & \lambda &= \frac{1}{2\sqrt{2J}} \frac{-\cos \theta + i(1 + \sin^2 \theta)}{(1 - i \cos \theta)^2 \sin \theta}, \\ \pi &= \frac{1}{\sqrt{2J}} \frac{i \sin \theta}{(1 - i \cos \theta)^2}. \end{aligned} \quad (4.34)$$

It is straightforward but tedious to compute the perturbations from the wave eigenfunctions Ψ^s .

Note that in determining the retarded Green's function from gravity, it is the asymptotic behaviours of the source and the response that matter. In other words, once the source term of the field is decided, its field strength has the same Gamma function dependence, up to a factor. Even though the Ψ^s 's are related to the field strength, the source term is still proportional to B^s , at most up to a factor which plays no essential role. Once the relative normalization between A^s and B^s is fixed, it is safe to take B^s as the source.

There is another tricky point. Actually the response to the source B^s cannot be taken as the A^s term in the same Ψ^s . Instead, the response should be given by the A^{-s} term in Ψ^{-s} . For example, for the fermionic perturbations, it has been shown that the conjugate momentum to ψ^- is proportional to ψ^+ [33]. And in the study of warped AdS/CFT correspondence, it was found that for vector perturbation, the conjugate momentum of A_i is not in itself, but in another component of A_μ [18]. In fact, for the gauge field, the conjugate momentum of A_μ is the field strength $F^{r\mu}$, which is related to Ψ^s directly. Here, even if we have no rigorous derivation, taking into account of above two points we would like to propose the following working prescription for computing the retarded Green's function for general perturbations with spin s :

$$G_R^s \sim (-1)^s \frac{A^{-s}}{B^s}. \quad (4.35)$$

Here $(-1)^s$ is mainly for the fermionic perturbations.

With this prescription, the retarded Green's function of the perturbations with spin s is given by

$$\begin{aligned} G_R^s &\sim (-1)^s \frac{A^{-s}}{B^s} \\ &\sim (-1)^s (2\alpha)^{2\beta} \frac{\Gamma(-2\beta) \Gamma(\frac{1}{2} + \beta - s - im) \Gamma(\frac{1}{2} + \beta - i\frac{\omega}{\alpha})}{\Gamma(2\beta) \Gamma(\frac{1}{2} - \beta + s - im) \Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})}. \end{aligned} \quad (4.36)$$

To get the second line, we have used the relation (4.22). With the conformal weights of the field identified as

$$h_R^s = \frac{1}{2} + \beta, \quad h_L^s = h_R^s - s, \quad (4.37)$$

the retarded Green's function can be rewritten as

$$G_R^s \sim (-1)^s T_R^{2h_R^s-1} \frac{\Gamma(1-2h_R^s)\Gamma(h_L^s-im)\Gamma(h_R^s-i\frac{\omega}{\alpha})}{\Gamma(2h_R^s-1)\Gamma(1-h_L^s-im)\Gamma(1-h_R^s-i\frac{\omega}{\alpha})}. \quad (4.38)$$

It is straightforward to check that at the Matsubara frequencies, the above retarded Green's function agrees precisely, up to a frequencies independent normalization factor, with the CFT result (3.9) if the frequencies and the temperatures are identified as before:

$$\omega_L = m, \quad \omega_R = \omega, \quad T_L = \frac{1}{2\pi}, \quad T_R = T_R. \quad (4.39)$$

The cross section can be read directly from the Green's function by the relation $\sigma \sim \text{Im}(G_R)$. It turns out that for the fermion, the cross section is

$$\sigma \sim \frac{(2\alpha)^{2h_R-1}}{\Gamma(2h_R-1)^2} \cosh((m + \frac{\omega}{\alpha})\pi) |\Gamma(h_L-im)\Gamma(h_R-i\frac{\omega}{\alpha})|^2. \quad (4.40)$$

And for the gauge field and the graviton, the cross sections are in the same form as

$$\sigma \sim \frac{(2\alpha)^{2h_R-1}}{\Gamma(2h_R-1)^2} \sinh((m + \frac{\omega}{\alpha})\pi) |\Gamma(h_L-im)\Gamma(h_R-i\frac{\omega}{\alpha})|^2. \quad (4.41)$$

They agree with the CFT result.

As for the quasi-normal modes, their frequencies are simply

$$\begin{aligned} \omega_L^s &= -i2\pi T_L(h_L^s + n_L) \\ \omega_R^s &= m\Omega_H - i2\pi T_R(h_R^s + n_R) \end{aligned} \quad (4.42)$$

with n_L, n_R being non-negative integers.

5. Real-time Correlators in NHEK

It is interesting to generalize the discussion in the last section to the NHEK geometry. At first look, the NHEK geometry is like the vacuum with the near-NHEK geometry as excitation. However, since NHEK itself is dual to the 2D chiral CFT with a temperature in the left-moving sector, it is thus more natural to take NHEK as a limiting case of near-NHEK.

To discuss the perturbations scattering in NHEK geometry, it is not appropriate to take the $\alpha \rightarrow 0$ limit since it leads to singularity in the eigenfunctions. To simplify the discussion, let us consider the massless perturbations so that we can treat all kinds of perturbations at the same time. The NP null tetrad could be taken as the $\alpha \rightarrow 0$ limit of the one in near-NHEK. We start with the same ansatz

$$\Phi^s = e^{-i\omega t + im\phi}(r)^{-2s} \mathcal{R}^s(r) \mathcal{S}^s(\theta). \quad (5.1)$$

The angular function $\mathcal{S}^s(\theta)$ satisfies the same equation (4.21), while the radial function obeys

$$\frac{d}{dr} \left(r^2 \frac{d}{dr} \right) \mathcal{R}^s(r) - \left(\Lambda_{lm} + q^2 - 2m^2 - \frac{(\omega + qr)^2}{r^2} \right) \mathcal{R}^s(r) = 0. \quad (5.2)$$

where

$$q = m - is. \quad (5.3)$$

Introduce $z = 1/r$, we can rewrite this equation into

$$\frac{d^2}{dz^2} \mathcal{R}^s + \left(\frac{\frac{1}{4} - \beta^2}{z^2} + \frac{2q\omega}{z} - \omega^2 \right) \mathcal{R}^s = 0, \quad (5.4)$$

where β is the same as defined in (4.26). The solution could be written in terms of Kummer function

$$\mathcal{R}_{\pm}^s = e^{-i\omega z} (2i\omega z)^{\frac{1}{2} \pm \beta} F\left(\frac{1}{2} \pm \beta - iq, 1 \pm 2\beta; 2\omega z\right). \quad (5.5)$$

We take the point of view that the horizon is at $r = 0$, and requires the ingoing boundary condition at the horizon. Then we find that the eigenfunction is the combination of the above two functions:

$$\mathcal{R}^s = A_+^s \mathcal{R}_+^s + A_-^s \mathcal{R}_-^s, \quad (5.6)$$

where

$$A_+^s = -\frac{\Gamma(1-2\beta)}{\Gamma(\frac{1}{2}-\beta-iq)} A_0, \quad A_-^s = \frac{\Gamma(1+2\beta)}{\Gamma(\frac{1}{2}+\beta-iq)} A_0, \quad (5.7)$$

with A_0 being a constant.

Note that our discussion on the perturbations is different from the one in [41], in which the treatment of NHEK geometry was in global coordinate. We would like to emphasize that the coordinate transformation from NHEK to global coordinate is singular. It is essential to take NHEK geometry as an extreme black hole such that its correspondence to 2D CFT at finite temperature is transparent.

Following the prescription, we have the retarded Green's function

$$\begin{aligned} G_R^s &\sim (-1)^s \frac{A_+^{-s}}{A_-^s} \\ &= (-1)^s \frac{\Gamma(-2\beta)\Gamma(\frac{1}{2}+\beta-s-im)}{\Gamma(2\beta)\Gamma(\frac{1}{2}-\beta+s-im)} \end{aligned} \quad (5.8)$$

$$= (-1)^s \frac{\Gamma(1-2h_R)\Gamma(h_L-im)}{\Gamma(2h_R-1)\Gamma(1-h_L-im)} \quad (5.9)$$

where we have identified

$$h_R = \frac{1}{2} + \beta, \quad h_L = h_R - s. \quad (5.10)$$

It is easy to see that the Green's functions are in consistence with the prediction of a CFT with only left-moving modes of frequency

$$\omega_L = m. \quad (5.11)$$

Therefore the NHEK background looks like an extreme black hole, corresponding to a “chiral” CFT at $T_L = 1/2\pi$ [18]. The cross sections follow from the retarded Green's function. For the fermionic perturbation, we have

$$\sigma \sim \frac{\Gamma(2(1-h_R))}{\Gamma(2h_R)} \cos(h_R\pi) \cosh(m\pi) |\Gamma(h_L-im)|^2, \quad (5.12)$$

while for the bosonic perturbation, we have

$$\sigma \sim \frac{\Gamma(2(1-h_R))}{\Gamma(2h_R)} \cos(h_R\pi) \sinh(m\pi) |\Gamma(h_L - im)|^2. \quad (5.13)$$

They are in agreement with the CFT results. The retarded correlation has the simple poles at

$$m = -i2\pi T_L(n_L + h_L). \quad (5.14)$$

Note that now the poles have nothing to do with the frequencies, so there is no quasi-normal mode for extreme Kerr black hole.

6. Kerr-Newman Case

For the Kerr-Newman black hole with mass M , angular momentum $J = aM$ and electric charge Q , its metric takes the following form

$$ds^2 = -\frac{\Delta}{\rho^2} (d\hat{t} - a \sin^2 \theta d\hat{\phi})^2 + \frac{\rho^2}{\Delta} d\hat{r}^2 + \rho^2 d\theta^2 + \frac{1}{\rho^2} \sin^2 \theta \left(a d\hat{t} - (\hat{r}^2 + a^2) d\hat{\phi} \right)^2, \quad (6.1)$$

where

$$\begin{aligned} \Delta &= (\hat{r}^2 + a^2) - 2M\hat{r} + Q^2, \\ \rho^2 &= \hat{r}^2 + a^2 \cos^2 \theta. \end{aligned} \quad (6.2)$$

There are two horizons located at

$$\hat{r}_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2}. \quad (6.3)$$

And the Hawking temperature, entropy, angular velocity of the horizon and the electric potential are respectively

$$\begin{aligned} T_H &= \frac{\hat{r}_+ - \hat{r}_-}{4\pi(\hat{r}_+^2 + a^2)}, \\ S &= \pi(\hat{r}_+^2 + a^2), \\ \Omega_H &= \frac{a}{\hat{r}_+^2 + a^2}, \\ \Phi &= \frac{Q\hat{r}_+}{\hat{r}_+^2 + a^2}. \end{aligned} \quad (6.4)$$

For the extreme case, $\hat{r}_+ = \hat{r}_-$ such that $T_H = 0$. However, from the first law of thermodynamics, we have nonvanishing left-moving temperature and left-moving chemical potential

$$T_L = \frac{\hat{r}_+^2 + a^2}{4\pi J}, \quad \mu_L = -\frac{Q^3}{2J}. \quad (6.5)$$

The near horizon geometry of extreme Kerr-Newman (NHEK-Newman) could be obtained by the same scaling limit as the one in extreme Kerr geometry [5, 37]. It is given by

$$ds^2 = \Gamma(\theta) \left(-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 \right) + \Lambda(\theta) (d\phi + br dt)^2, \quad (6.6)$$

where

$$\begin{aligned}\Gamma(\theta) &= \hat{r}_+^2 + a^2 \cos^2 \theta \\ \Lambda(\theta) &= \frac{(\hat{r}_+^2 + a^2)^2 \sin^2 \theta}{\hat{r}_+^2 + a^2 \cos^2 \theta}, \\ b &= \frac{2a\hat{r}_+}{\hat{r}_+^2 + a^2}.\end{aligned}\tag{6.7}$$

The gauge potential and the field strength are

$$\begin{aligned}A &= \frac{Q}{b} \frac{\hat{r}_+^2 - a^2 \cos^2 \theta}{\hat{r}_+^2 + a^2 \cos^2 \theta} (d\phi + brdt), \\ F &= -Q \frac{\hat{r}_+^2 - a^2 \cos^2 \theta}{\hat{r}_+^2 + a^2 \cos^2 \theta} (d\phi + brdt) dt \wedge dr + Q \frac{(\hat{r}_+^2 + a^2) \hat{R}_+ a \sin \theta \cos \theta}{(\hat{r}_+^2 + a^2 \cos^2 \theta)^2} d\theta \wedge (d\phi + brdt).\end{aligned}\tag{6.8}$$

For the near-extremal Kerr-Newman black hole, the modes in the right-moving sector are excited. Taking a scaling limit with a finite right-moving temperature, we obtain the near-NHEK-Newman geometry

$$ds^2 = \Gamma(\theta) \left(-r(r + 2\alpha) dt^2 + \frac{dr^2}{r(r + 2\alpha)} + d\theta^2 \right) + \Lambda(\theta) (d\phi + b(r + \alpha) dt)^2, \tag{6.9}$$

with $\alpha = 2\pi T_R$. Similar to the Kerr black hole case, the near-NHEK-Newman geometry looks like a black hole with horizon at $r = 0$, while NHEK-Newman geometry is an extreme black hole with the horizon at $r = 0$.

For simplicity, we will focus on the complex scalar field with mass μ and charge e . The Klein-Gordon equation is

$$(\nabla_\mu + ieA_\mu)(\nabla^\mu + ieA^\mu)\Phi - \mu^2\Phi = 0. \tag{6.10}$$

With the ansatz (4.5), the angular wave function satisfies

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d}{d\theta} \mathcal{S} \right) + \left(\Lambda_{lm} - a^2(\omega_0^2 - \mu^2) \sin^2 \theta - \frac{m^2}{\sin^2 \theta} \right) \mathcal{S} = 0. \tag{6.11}$$

Here Λ_{lm} is the separation constant. It is restricted by the regularity boundary condition at $\theta = 0, \pi$ and can be computed numerically. ω_0 is the frequency saturate the superradiant bound

$$\omega_0 = m\Omega_H + e\Phi. \tag{6.12}$$

The radial part \mathcal{R} satisfies the equation

$$\frac{d}{dr} \left(r(r + 2\alpha) \frac{d}{dr} \right) \mathcal{R}(r) - \left(\Lambda_{lm} + \mu^2(\hat{r}_+^2 + a^2) - 2am\omega_0 - \frac{(\omega + \tilde{m}(r + \alpha))^2}{r(r + 2\alpha)} \right) \mathcal{R}(r) = 0, \tag{6.13}$$

where

$$\begin{aligned}\tilde{m} &= bm + \frac{\hat{r}_+^2 - a^2}{\hat{r}_+^2 + a^2} eQ \\ &= \frac{m - e\mu_L}{2\pi T_L}\end{aligned}\tag{6.14}$$

The radial eigenfunction which is ingoing at the horizon $r = 0$ is

$$\mathcal{R}(r) = r^{-\frac{i}{2}(\tilde{m} + \frac{\omega}{\alpha})} \left(\frac{r}{2\alpha} + 1 \right)^{\frac{i}{2}(\frac{\omega}{\alpha} - \tilde{m})} F\left(\frac{1}{2} + \beta - i\tilde{m}, \frac{1}{2} - \beta - i\tilde{m}, 1 - i(\tilde{m} + \frac{\omega}{\alpha}); -\frac{r}{2\alpha}\right), \quad (6.15)$$

where

$$\beta^2 = \frac{1}{4} + \Lambda_{lm} - 2am\omega_0 - \tilde{m}^2 + \mu^2(M^2 + a^2). \quad (6.16)$$

Asymptotically, the radial eigenfunction could be expanded as

$$\mathcal{R}(r) \sim Ar^{-\frac{1}{2}-\beta} + Br^{-\frac{1}{2}+\beta}, \quad (6.17)$$

where

$$A = N \frac{\Gamma(-2\beta)\Gamma(1 - i(\tilde{m} + \frac{\omega}{\alpha}))}{\Gamma(\frac{1}{2} - \beta - i\tilde{m})\Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})} (2\alpha)^{\frac{1}{2}+\beta-\frac{i}{2}(\tilde{m} + \frac{\omega}{\alpha})} \quad (6.18)$$

$$B = A(\beta \rightarrow -\beta), \quad (6.19)$$

with N being a constant. Compared to the near-NHEK case, there are two differences. One is the value of β . However the conformal weight of the scalar is still

$$h_R = \frac{1}{2} \pm \beta, \quad (6.20)$$

so this difference does not affect the discussion. The other difference is on the angular momentum dependence. Now we have \tilde{m} , which has been shifted by the chemical potential in left-moving sector.

The retarded Green's function is

$$\begin{aligned} G_R &\sim \frac{A}{B} \\ &= \frac{\Gamma(-2\beta)}{\Gamma(2\beta)} \frac{\Gamma(\frac{1}{2} + \beta - i\tilde{m})\Gamma(\frac{1}{2} + \beta - i\frac{\omega}{\alpha})}{\Gamma(\frac{1}{2} - \beta - i\tilde{m})\Gamma(\frac{1}{2} - \beta - i\frac{\omega}{\alpha})} (2\alpha)^{2\beta}. \end{aligned} \quad (6.21)$$

Taking the following identification into account,

$$h_L = h_R = \frac{1}{2} + \beta, \quad \omega_L = m - e\mu_L, \quad \bar{\omega}_R = \omega, \quad T_L = \frac{M^2 + a^2}{4\pi Ma}, \quad T_R = T_R, \quad (6.22)$$

the retarded Green's function and the cross section are both in good agreement with the CFT prediction. Note that due to the coupling with the background electric field, the angular quantum number m gets shifted. In other words, the presence of the electric field affect only the left-moving sector.

Similarly, the quasi-normal modes could be read from the poles in the retarded Green's function. These poles are located at

$$\begin{aligned} \tilde{m} &= -i(n_L + h_L) \\ \omega &= -i2\pi T_R(n_R + h_R), \end{aligned} \quad (6.23)$$

with n_L, n_R being non-negative integers. The second relation gives the quasi-normal modes

$$\omega_R = m\Omega_H - i2\pi T_R(n_R + h_R), \quad (6.24)$$

after taking into account of the superradiant bound.

For the case of NHEK-Newman geometry, the analysis is very similar to the NHEK case. The radial eigenfunction of complex massive scalar could be written in terms of Kummer function. And the retarded Green's function and the cross section are of the same form as (4.13) and (4.15) with m being replaced by \tilde{m} . This suggests that the NHEK-Newman geometry is dual to a chiral part of 2D CFT with a chemical potential and a non-vanishing left temperature T_L .

7. Conclusions and Discussions

In this paper, we studied the real-time correlators in Kerr/CFT correspondence. Kerr/CFT correspondence states that a quantum gravity theory in NHEK geometry is dual to chiral (left) part of a 2D CFT with left-moving temperature. For near-extremal Kerr black holes, there are excited right-moving modes as well, which is dual to the right-moving excitations in dual 2D CFT. Remarkably, this phenomenon shows up nicely in a geometrical way, fitting well with the warped AdS/CFT correspondence. In fact, NHEK geometry is an extreme warped AdS₃ black hole for fixed θ , whose dual CFT has only non-vanishing left-moving temperature; while near-NHEK geometry is a non-extremal AdS₃ black hole for fixed θ , whose dual CFT has both non-vanishing left- and right-moving temperatures. Such a picture has been discussed in the warped AdS₃/CFT correspondence in [18]. Using the fact that the NHEK and the near-NHEK geometry can be considered as a warped AdS geometry, we applied the holographic prescription in AdS/CFT correspondence and computed the retarded correlators, the cross sections of various perturbations and the quasi-normal modes in these geometries. Perfect agreement with the CFT predictions was found. We have also generalized the discussion to the cases of extreme Kerr-Newman and the near extreme Kerr-Newman. The picture is similar and we found a new effect in the shift of the frequency due to the existence of an electric field.

Some of the results in this paper, including the cross-sections of various perturbations in near-NHEK, near-NHEK-Newman have been obtained in [7, 15]. Here we have presented a different derivation using the retarded Green's functions. By considering the Kerr/CFT correspondence as a warped AdS/CFT correspondence, we emphasize the possibility and necessity to consider the perturbations entirely in near-horizon geometries. In particular, we investigated the perturbations in NHEK and NHEK-Newman geometries and showed that they are extreme black holes dual to only the left-parts of 2D CFTs. One interesting question we would like to ask is what corresponds to a 2D CFT without temperature.

Our treatment could also be applied to the multi-charged and higher-dimensional black holes as discussed in [16]. Even though we are going to work in higher dimension, the extra isometries will simplify the analysis and allow us to study two decomposed master equations, similar to what we have met. The key point is still that the master radial equation suggests that the perturbation is scattering off a warped black holes. We will leave the details.

Despite many successful checks of the Kerr/CFT correspondence, the subject is far from understood and its consistency has been challenged, particularly concerning the GHSS

boundary conditions [3]. Recently other boundary conditions which enhance the $SL(2, R)_R$ isometry to Virasoro in addition to the left-moving Virasoro algebra have been proposed [38, 39]. Due to their specific form, it is not clear whether these generators actually describe a symmetry or just gauge transformation [40]. The dynamics of the near horizon geometry subjected to the GHSS boundary conditions has been examined [41, 42] and it has been argued that the NHEK geometry contains no dynamics. It is important to understand better these issues.

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